10-2 day 1 Vector's in the Plane

Learning Objectives:

I can find the magnitude and direction of a vector in component form

I can find the x and y components of a vector in polar form

I can perform basic vector operations

I can find unit vectors

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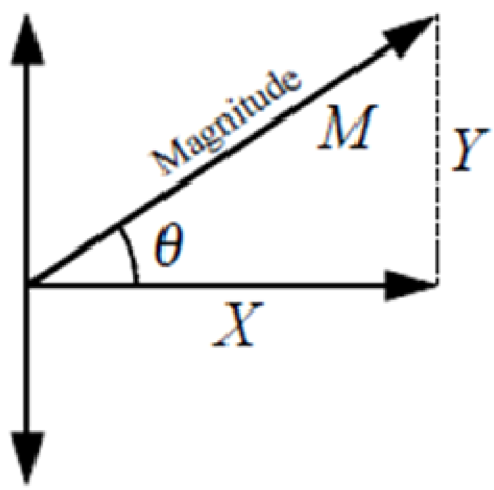
Vectors have both a magnitude and direction. Vectors can be expressed using x and y components <x, y> (rectangular, component or Cartesian form) or using the direction and magnitude $(r;\theta)$ (Polar Form).



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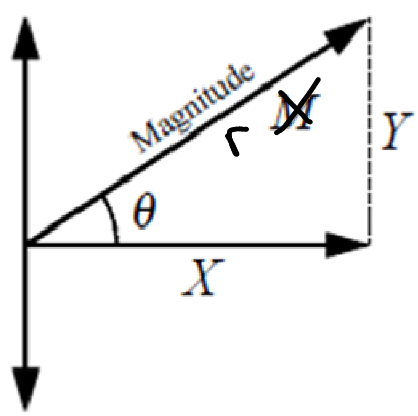
To find the x and y components of a vector with magnitude r and direction θ is



$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$

$\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta$

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Ex1. A force of 6N is acting at 78° to the x-axis. Express this vector in component (rectangular or Cartesian) form.

$x = 6\cos 78° = 1.247$

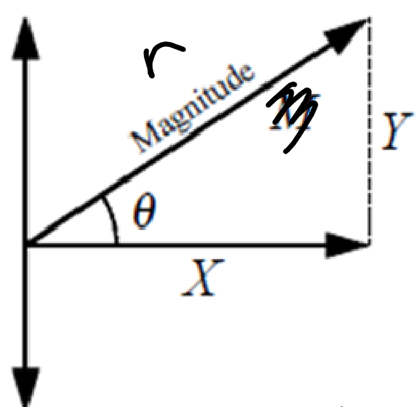
$y = 6\sin 78° = 5.868$

$\vec{v} = \langle 1.247, 5.868 \rangle$

To find the magnitude and direction of vector in component form <x, y>

$\langle x, y \rangle$

$(r; \theta)$?



$x^2 + y^2 = r^2$

$r = \sqrt{x^2 + y^2}$

$\tan\theta = \frac{y}{x}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

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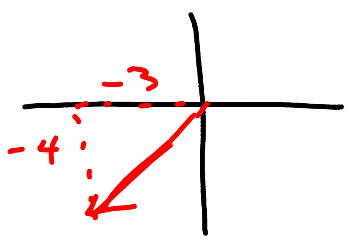
Ex2. Find the magnitude and direction of <-3, -4>



$\tan^{-1}\left(\frac{4}{3}\right) = 53^\circ$

$+180$

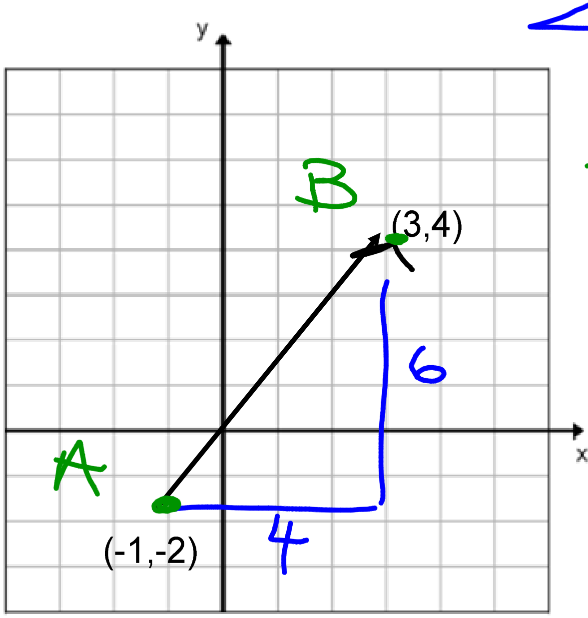
233°

$\sqrt{(-3)^2 + (-4)^2} = 5$

$(5, 233^\circ)$

Ex3. Find the magnitude and direction of the vector below:

$\langle 4, 6 \rangle$



$\overrightarrow{AB} = \langle 3 - {}^-1, 4 - {}^-2 \rangle$

$\overrightarrow{AB} = B - A$

To find the component form of a vector with tail at A(x_1, y_1) and head at B(x_2, y_2) is

$$\overrightarrow{AB} = \langle x, y \rangle = \langle x_2 - x_1, y_2 - y_1 \rangle$$

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Vector Operations

Vector Addition

$$< x_1, y_1 > + < x_2, y_2 > = < x_1 + x_2, y_1 + y_2 >$$

Scalar Multiplication

$$k < x, y > = < kx, ky >$$

Opposite of a Vector

If $\bar{v} = < x, y >$, then the opposite of $\bar{v}$ is

$$-\bar{v} = < -x, -y >$$

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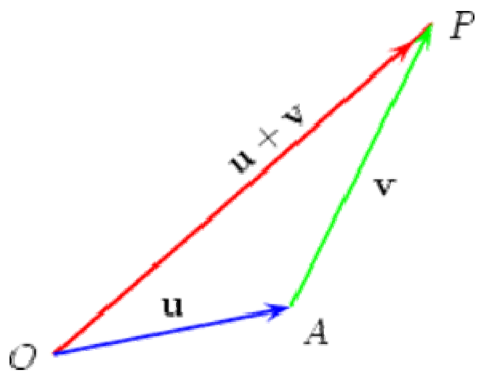
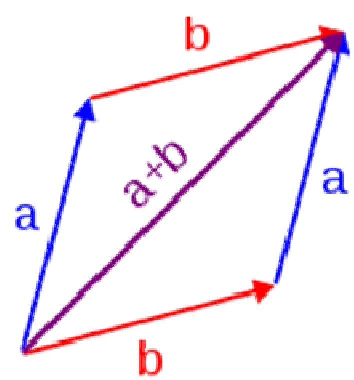
Geometric Interpretation of Vectors

Tip to Tail Method



Parallelogram Method



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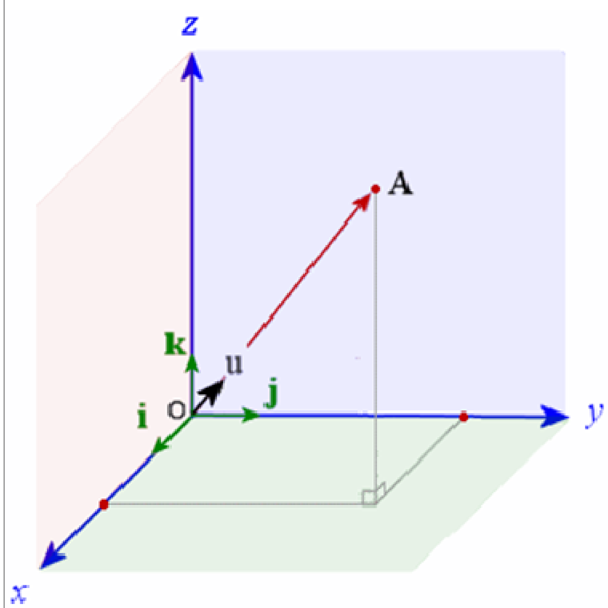
Unit Vectors



A **unit vector** is a vector that is 1 unit long. Typically used to show a direction without a magnitude.

$$\bar{v} = \langle x, y \rangle$$

$$\frac{\bar{v}}{|\bar{v}|} = \frac{\langle x, y \rangle}{\sqrt{x^2 + y^2}}$$

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Ex4. Write the vector $\bar{u} = <-3, 8>$ as a unit vector.

$r = \sqrt{(-3)^2 + (8)^2}$

$r = \sqrt{9 + 64}$

$r = \sqrt{73}$

$\left< \frac{-3}{\sqrt{73}}, \frac{8}{\sqrt{73}} \right>$

Properties of Vectors

Let $\vec{u}$ and $\vec{v}$ be vectors and let a and b be scalers.

1.) $\vec{u}+\vec{v}=\vec{v}+\vec{u}$ Commutative Property

2.) $\vec{u}+0=\vec{u}$ Zero Property of Addition

3.) $0\cdot\vec{u}=0$ Zero Property of Mult

4.) $\left(\vec{u}+\vec{v}\right)+\vec{w}=\vec{u}+\left(\vec{v}+\vec{w}\right)$ Associative Property

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5.) $\vec{u} + -\vec{u} = 0$ Inverse Property

6.) $a \cdot \left(b\vec{u}\right) = \left(ab\right) \cdot \vec{u}$ Associative Property of Scalar Multiplication

7.) $a \cdot \left(\vec{u} + \vec{v}\right) = a \cdot \vec{u} + a \cdot \vec{v}$ Distributive Property

8.) $\left(a + b\right) \cdot \vec{u} = a \cdot \vec{u} + b \cdot \vec{u}$ Distributive Property

9.) $1 \cdot \vec{u} = \vec{u}$ Identity Property of Mult

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Ex5. Given $\vec{a} = \langle 2, -7 \rangle$ and $\vec{b} = \langle -4, 2 \rangle$. Find:

a.) $\vec{a} + \vec{b}$

b.) $3\vec{b}$

C.) $2\vec{a} - 4\vec{b}$

d.) $\left|\frac{1}{4}\vec{b}\right|$

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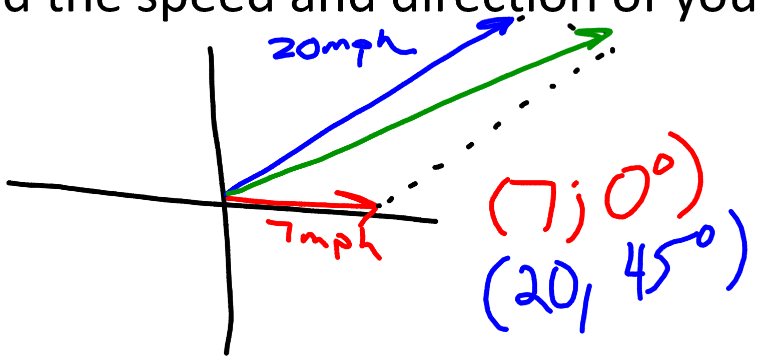
Ex6. You are paddling a canoe across a lake. You are trying to paddle due East but there is wind coming out of the Southwest at 20 mph which is blowing you off course. If you are paddling at 7 mph, find the speed and direction of your canoes path.



$(7, 0°)$ $(20, 45°)$ $y = r\sin\theta$ $x = r\cos\theta$

$\langle 7, 0\rangle + \langle 10\sqrt{2}, 10\sqrt{2}\rangle$ $\langle 7+10\sqrt{2}, 10\sqrt{2}\rangle$ $r = \sqrt{(7+10\sqrt{2})^2+(10\sqrt{2})^2} = 25.436$ mph

$\theta = \tan^{-1}\left(\frac{10\sqrt{2}}{7+10\sqrt{2}}\right) = 33.779°$ $(25.436; 33.779°)$

N of E

Homework

pg 545 # 3, 5, 6, 8, 12, 13, 15, 17-22, 25, 26

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